Note: Half-Space Triangle Rasterization

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#### Cross Product (Perp Dot Product)

(1)

* is also written as , means “perp dot product”.
  + , , are parallel.
  + , is counterclockwise from .
  + , is counterclockwise from .
* the direction of cross product obeys “right hand rule”.
* the magnitude of cross product equals the area of a parallelogram
* with vectors and .



* Also can see [1].

#### Edge function[2]



* If the vertices , , are given in counterclockwise order, then

“ is inside the triangle ”

is equivalent to

* According to the formula [1], can be described as:

(2)

Perform the multiplications and by rearranging the factors, the above formula can be written as the “edge function”:

(3)

* the other two edge functions are the following:

(4)

(5)

* For convenience of programming, this three edge functions are numbered as following:

(6)

(7)

(8)

It is easy to find out that:

(9)

(10)

The same as the other two edge functions.

* Then, we can iterate triangle as the following pseudo code



#### Barycentric Interpolation



* The properties (attributes) of the point can be calculated by the barycentric coordiantes. includes depth, normal, color, texture coordinates, etc.

(11)

(12)

And, if is inside the triangle,

* According to formulas (1) and (6),

(13)

the other two coefficients are the following:

(14)

(15)

Also can see [3].

#### Perspective-Correct Interpolation[4]

* Barycentric coordinates are not invariant under projection, it can not produce the perspective correct result. we should use the interpolation to get the correct z-depth as the following:

(16)

According to formulas (13,14,15) and rearranging the factors, the above formulas can be described as:

(17)

Then,

For convenience, written as:

(18)

And it easy to will find out that:

(19)

(20)

(21)

(22)

So the perspective correct z-depth can also use for iteration-based algorithm.

* Now there is a vector , if divided by (the value is ) , the is given. It is friendly to CPU/GPU calculation, due to memory alignment and SIMD fitting.
* Additionally, use the interpolation to get other correct attributes as the following:

(23)

* The pseudo code is the following:



#### References

[1] *vectors cross product*[DB/OL]

https://www.mathsisfun.com/algebra/vectors-cross-product.html

[2] Mileff P , K Nehéz , Dudra J . *Accelerated Half-Space Triangle*

*Rasterization*[J]. Acta Polytechnica Hungarica, 2015, 12(7):2015-2217.

[3] *Barycentric coordinate system*[DB/OL]

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[4] Low K L . *Perspective-Correct Interpolation*[J].

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